The gravitational field & the vacuum energy field

As we know from classical physics, the amount of gravitational field energy of a radius one million light year of space for the Sun should be the sum of the potential energy of the field (thus an integral of the potential energy of the Sun from the surface (Rsun) to $9.461\cdot 10^{15}$ m). The gravitational field strength at a distance r is $-G\cdot M/r^2$, and the energy is

$$E = \int_{Rsun}^{MLY} -\frac{GM}{r^2} r dV = \int_{Rs}^{MLY} -\frac{GM}{r^2} r 4\pi r^2 dr = \int_{Rsun}^{MLY} -4\pi GM\, r dr = -(2\pi GM)(MLY^2 - Rsun^2)$$

The energy is negative because the field is conservative. In superposition with all the stars in the Milky Way, we can add the energy of all the stars, supposing that the holes of stars have the same radius Rsun and are negligible.
We have the approximate formula $E = -2\pi G\, Mmw \cdot 9.461^2 \cdot 10^{42}$ in the Newtonian paradigm, the energy of the Milky Way within a radius of 1 million light years & for R=4.12million LY we have the gravitational energy.
$Eg \sim -8.212653398\cdot 10^{77}$ J & absolute energy density $3.2479\cdot 10^9$ J/m^3.
For the onduscular equation of gravity within a radius of $2R = 2\cdot 3.9228\cdot 10^{22}$ m, we have to substitute the force of Newtonian attraction with generalized gravity, thus we have:

$$E = \int_{Rsun}^{2R} -\frac{GMmw}{1.0496\cdot 10^{44}}\left(\frac{1.0496\cdot 10^{44}}{r^2} - \frac{3.12166\cdot 10^{21}}{r} + 0.02274\right) r dV$$
$$= \int_{Rs}^{2R} -\frac{GMmw}{1.0496\cdot 10^{44}}\left(\frac{1.0496\cdot 10^{44}}{r^2} - \frac{3.12166\cdot 10^{21}}{r} + 0.02274\right) r 4\pi r^2 dr$$
$$= 4\int_{Rs}^{2R} -\frac{GMmw}{1.0496\cdot 10^{44}}(1.0496\cdot 10^{44} - 3.12166\cdot 10^{21} r + 0.02274 r^2) r\pi dr \sim -7\cdot 10^{123} J$$

With absolute energy density = $4.3259\cdot 10^{53}$ J/m^3.
The energy of vacuum is calculated as: $E = \hbar/2\cdot Tq$, where Tq is Time Quanta, the time that light travels to radius Bohr $\sim 1.756\cdot 10^{-19}$ sec, $E_{0vac} = -2.9874555\cdot 10^{-16}$ J, so the density of energy absolute value is
DEvac= $4.8111\cdot 10^{14}$ J/m^3.
The flux equation for μ neutrino considering spherical coordinates symmetry & the speed of graviton equal to light speed is for (electronic neutrino), the second-order flux equation wave equation:
$\frac{\partial^2 P(r,t)}{c^2 \partial t^2} - \frac{\partial^2 P(r,t)}{\partial r^2} = 0$ With initial condition $P(2R_s,t)=0$ & diff(P(0,0),r)= $-R_s = -7.766461840\cdot 10^{25}$ m (8.209331978billion lightyears); Thus the solution of Λ Space flux/ surface unit $\Lambda flux = P(r,t)/4\pi r^2$ equation (23):

$$\frac{P(r,t)}{4\cdot\pi\cdot r^2} = \frac{(\_c_2\cdot t^2 + 4\cdot R_s^2 - 2\cdot R_s\cdot r + 2\_C3\cdot t)\cdot c^2 + (-4\cdot R_s^2 + r^2)\_c_2}{8\cdot\pi\cdot r^2\cdot c^2}$$

And with initial condition t=0 (24):

$$\frac{P(r,t)}{4\cdot\pi\cdot r^2} = \frac{(4\cdot R_s^2 - 2\cdot R_s\cdot r)\cdot c^2 + (-4\cdot R_s^2 + r^2)\_c_2}{8\cdot\pi\cdot r^2\cdot c^2}$$

$\int_0^{2R_s} \frac{P(r,t)}{4\cdot\pi\cdot r^2} dr = \int_0^{2R_s} \left(\frac{(4\cdot R_s^2 - 2\cdot R_s\cdot r)\cdot c^2 + (-4\cdot R_s^2 + r^2)\_c_2}{8\cdot\pi\cdot r^2\cdot c^2}\right) dr = 5.725\cdot R_s/\dim(R_s)$ The total space growth of about 47 billion LY and 47/8.209 =5.725 thus:
$\_c_2 = -9.711452983c^2$ Thus, we have equations (25):

$$\Lambda flux = \frac{4\cdot R_s^2 - 4(-9.711452983)\cdot R_s^2}{8\cdot\pi}\cdot\frac{1}{r^2} - \frac{R_s}{4\cdot\pi}\cdot\frac{1}{r} + \frac{9.711452983}{8\cdot\pi}$$

For the vacuum energy field, we follow the same process starting from the flux:

$$\Lambda = \frac{4\cdot R_s^2 + 38.84581193\cdot R_s^2}{8\cdot\pi}\cdot\frac{1}{r^2} - \frac{R_s}{4\cdot\pi}\cdot\frac{1}{r} + \frac{9.711452983}{8\cdot\pi}$$

With r in m, we have:

$$\Lambda = 1.028288415 * 10\text{^}52\cdot\frac{1}{r^2} - 6.180353960 * 10\text{^}24\cdot\frac{1}{r} - 0.3864064366$$

Supposing that the radius of the Local Universe is 47 billion light-years due to the space expansion (space growth).
We can do the calculus of the exact radius LU backward from the equation of the density of energy equal to the Vacuum density Energy.
The Casimir force is the torsion force, the 5th force that exists and from the formula $F_{Casim} = \pi^2\cdot c\cdot\hbar A/(240\cdot L^4)$

And from the quark into muon neutrino we can write $F_C = \pi^3 \cdot c \cdot \hbar / (4 \cdot 240 \cdot Sq^2)$; where space cuanta $Sq = 4.3 \cdot 10^{-19}$ m.
Thus, Fc = 18.857 N is the Casimir force at the muon level.
$Ev = 4 \cdot \pi \cdot Rs^2 \int_{r0}^{2Rs} \Lambda \cdot Fc \, dr$ where Rs = 8.209331978 billion lightyears and Λ is the space growth.
Thus, the vacuum energy is: $Evac = 1.30686 \cdot 10^{79} \cdot 18.86 = 2.464 \cdot 10^{80}$ Joule, and the volume of the Local Universe is: $VolUL := 1.2962 \cdot 10^{78}$ m^3 so absolute density is $DEv = 3.1954 \cdot 10^2$ J/m^3

Calculus link: Egf.mw 30 June 2025