

## The modified Einstein-Planck law document

Planck's law should be modified so that the frequencies, respectively, wavelengths of 380.43nm and 3.08 nm have the same energy, 3.259eV. The initial suppositions, according <sup>[1]</sup> Landau & Lifshitz Vol 9 Statistical Physics Cap 5 Paragraph 63 Black-body Radiation (63.4) page 184 <http://www.michaelvio.byethost8.com/Landau&Lifsh.pdf>

1) The electromagnetic radiation of black-body radiation is in thermal equilibrium.

2) The photons do not interact with one another (the superposition principle), so the radiation may be regarded as a photon gas like an ideal gas.

3) The distribution of photons among the various quantum states with definite values of the momentum and energies  $\epsilon = \hbar\omega$  is given by Planck's distribution law of black body for photons of Bose statistics is (according to 63.3) <sup>[1]</sup>. The energy per quanta  $T_q$  is:  $E_{\nu q} = N(\nu) \cdot U(\nu, T)$ , (1) is the internal energy written  $U(\nu, T) = \frac{E_\nu}{e^{\frac{E_\nu}{kT}} - 1}$  and  $N(\nu) = \frac{8\pi\nu^2}{c^3} V$

is the number of states of the oscillators times the volume  $V$  of photonics gas  $T_q \cdot c \cdot \pi \cdot r_b^2$ ; where  $V$  is the volume of photonic gas  $\Rightarrow$  we consider a slightly variation of  $h$  depend on frequency  $E = \nu \cdot h(\nu)$ ; (2) and we will demonstrate a more general differential equation valid for any frequency (3).

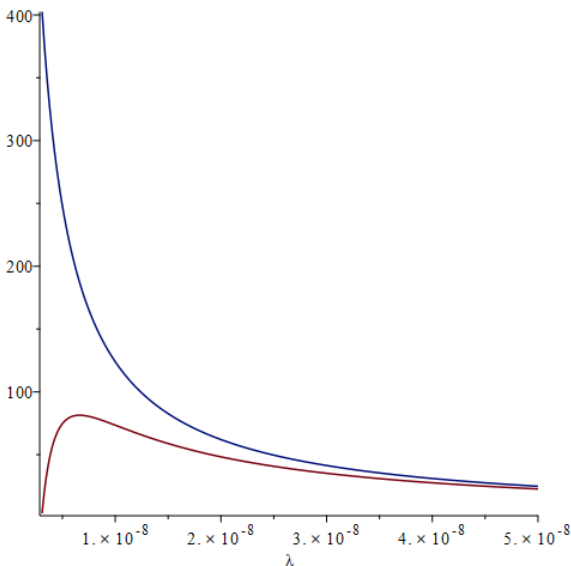
With  $\omega = 2\pi\nu$  and  $N(\nu) = \frac{8\pi V \nu^2}{c^3}$  And  $V$  is the quanta of photonic gas with cylindrical symmetry of the height of the cylinder is  $T_q \cdot c$ , and with radius  $r_b$  and area  $\pi \cdot r_b^2$  Thus, the volume ( $V$ ) of photic gas has the value  $T_q \cdot c \cdot \pi \cdot r_b^2 = \pi \cdot r_b^3 \Rightarrow$  The energy per quanta  $T_q$  is:  $E_{\nu q} = N(\nu) \cdot U(\nu, T)$ ; For the unit energy per time quanta per angle unit, where  $U(\nu, T)$

(or  $\langle E \rangle$ ) is the internal energy written  $U(\nu, T) = \frac{E_\nu}{e^{\frac{E_\nu}{kT}} - 1}$  and  $N(\nu) = \frac{8\pi\nu^2}{c^3} V$  is the number of states of the oscillators times the volume of photonics gas  $T_q \cdot c \cdot \pi \cdot r_b^2$ . Thus, for the quanta energy of elementary volume for a photon with velocity "c", period of oscillations  $t_0$ , and the sum of infinitesimal value  $t=1/\nu$  for  $T_q$  (quanta  $\Rightarrow T_q = 1.765 \cdot 10^{-19}$  sec;  $r_{Bohr} = c/T_q$ ) is:

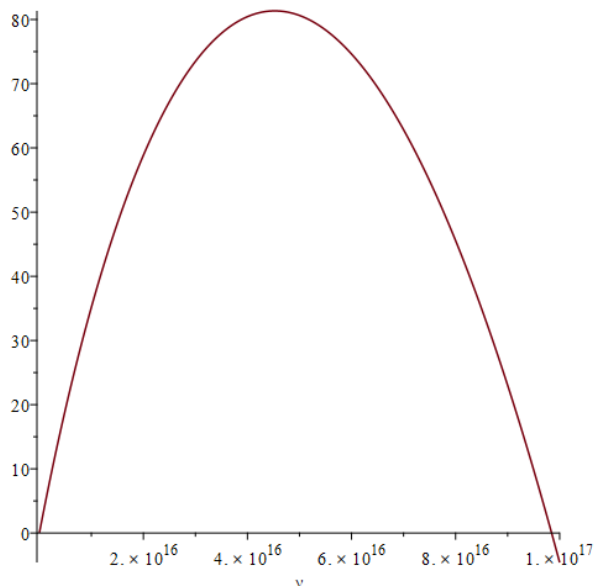
$$E_\nu = \frac{1}{T_q^2} \cdot \text{diff} \left( \frac{T_q \cdot c \cdot r_b^2 \cdot 8 \cdot \pi^2 \cdot \nu^2}{c^3} \frac{E_\nu}{e^{\frac{E_\nu + E_\mu}{E_0}} - 1}, \nu \right) = \text{diff} \left( T_q \cdot 8 \cdot \pi^2 \cdot \nu^2 \frac{E_\nu}{e^{\frac{E_\nu + E_\mu}{E_0}} - 1}, \nu \right) \quad (3)$$

Where  $E_\mu$  is the minimum of photon energy close to the energy of the muon neutrino

$E_\mu = 0.020\text{eV}$ , and  $E_0$  is the reference energy. Below the wavelength of 30 nm, the error becomes larger, and below 15nm, the error is dominant.



A plot of Energy theoretical and practical in [eV] versus lambda in [nm]



Graph of the Planck's constant [eV] as a function of lambda in [nm]

### We make the supposition:

1. Weak logarithmic variation:  $|\epsilon| \ll 1$  throughout the frequency range
2. Constant energy scales:  $E_\mu$  and  $E_0$  are constants independent of  $\nu$
3. Perturbative validity: Expansion valid when  $|\epsilon \ln(\nu/\nu_0)| \ll 1$
4. Thermodynamic interpretation: When  $E_0 = kT$  and  $E_\mu$  represents chemical potential
5. Modified dispersion: The photon energy relation  $E(\nu) = \nu \cdot h(\nu)$  with weak frequency dependence
6. First-order truncation: All  $O(\epsilon^2)$  and higher terms are neglected

So the energy of photons is proportional to the derivative of the Planck body law with respect to  $\nu$ , as in equation (3).

The interval  $\nu_{\min} \div \nu_{\max}$  is the lowest frequency of the photon energy, probably wavelength  $30\mu\text{m} \div 3\text{nm}$ .

Consideration on equation (3) => We start from this equation so  $h(\nu)$  thus Planck's constant slightly depend on  $\nu$ :  $E(\nu) = \nu h(\nu)$  Assume of small logarithmic variation:  $h(\nu) = h_0 \cdot [1 + \varepsilon \cdot \ln(\nu/\nu_0)]$  where  $\varepsilon \ll 1$  is a small dimensionless parameter and  $\nu_0$  is a reference frequency.

$$F(\nu) = T_q \cdot 8 \cdot \pi^2 \cdot \nu^2 \frac{E_\nu}{e^{\frac{E_\nu + E_\mu}{E_0}} - 1}$$

Define the denominator for convenience:  $D(\nu) = e^{\frac{E_\nu + E_\mu}{E_0}} - 1$

Then:  $F(\nu) = 8\pi^2 T_q \cdot \frac{\nu^2 E(\nu)}{D(\nu)}$

Applying the quotient rule:

$$\frac{dF(\nu)}{d\nu} = 8\pi^2 T_q \cdot \frac{d}{d\nu} \left[ \frac{\nu^2 E(\nu)}{D(\nu)} \right] = \frac{T_q \cdot 8 \cdot \pi^2}{D(\nu)^2} \left\{ D(\nu) \cdot \frac{d}{d\nu} [\nu^2 E(\nu)] - \nu^2 E(\nu) \frac{dD(\nu)}{d\nu} \right\}$$

**Numerator term 1:** Product rule on  $\nu^2 E(\nu)$ :

$$\frac{d}{d\nu} [\nu^2 E(\nu)] = 2\nu E(\nu) + \nu^2 \frac{dE}{d\nu}$$

**Numerator term 2:** Chain rule on  $D(\nu)$ :

$$\begin{aligned} \frac{dD(\nu)}{d\nu} &= \frac{d}{d\nu} \left[ \exp\left(\frac{E(\nu) + E_\mu}{E_0}\right) - 1 \right] = \exp\left(\frac{E(\nu) + E_\mu}{E_0}\right) \cdot \frac{1}{E_0} \cdot \frac{dE(\nu)}{d\nu} \\ &= \frac{[\exp((E(\nu) + E_\mu)/E_0)]}{E_0} \cdot \frac{dE(\nu)}{d\nu} = \frac{D(\nu) + 1}{E_0} \end{aligned}$$

**Combining:**

$$\begin{aligned} \frac{dF(\nu)}{d\nu} &= 8\pi^2 T_q \cdot \frac{D(\nu) \cdot [2\nu E(\nu) + \nu^2 \frac{dE(\nu)}{d\nu}] - \nu^2 E(\nu) \cdot \frac{(D(\nu) + 1)}{E_0} \frac{dE(\nu)}{d\nu}}{D(\nu)^2} \\ \frac{dF(\nu)}{d\nu} &= \frac{T_q \cdot 8 \cdot \pi^2}{D(\nu)^2} \left[ 2\nu E(\nu) \cdot D(\nu) + \nu^2 \frac{dE(\nu)}{d\nu} \left( \frac{D(\nu) - E(\nu)(D(\nu) + 1)}{E_0} \right) \right] \\ \frac{dF}{d\nu} &= \frac{8\pi^2 T_q \nu}{D^2} \left[ 2ED + \nu \frac{dE(\nu)}{d\nu} \left( \frac{D(\nu) - E(\nu)(D(\nu) + 1)}{E_0} \right) \right] \end{aligned}$$

Substituting:

- $E(\nu) \approx h_0 \nu [1 + \varepsilon \ln(\nu/\nu_0)]$
- $dE/d\nu \approx h_0 [1 + \varepsilon(1 + \ln(\nu/\nu_0))]$

Define dimensionless variables:

- $x = h_0 \nu / E_0$  (dimensionless frequency-energy ratio)
- $\mu = E_\mu / E_0$  (dimensionless chemical potential)
- $\ell = \ln(\nu/\nu_0)$  (logarithmic frequency ratio)

Then to first order in  $\varepsilon$ :

$$E(\nu) = E_0 x [1 + \varepsilon \ell]$$

$$D(\nu) \approx e^{x+\mu} [1 + x\varepsilon\ell] - 1 = (e^{x+\mu} - 1) + e^{x+\mu} x\varepsilon\ell$$

Expanding  $D$  to first order in  $\varepsilon$ :

$$D(\nu) \approx e^{x+\mu} [1 + x\varepsilon\ell] - 1 = (e^{x+\mu} - 1) + e^{x+\mu} x\varepsilon\ell$$

Let  $D_0 = e^{x+\mu} - 1$  (the zeroth-order term). Then:

$$D(\nu) \approx D_0 + (D_0 + 1)x\varepsilon\ell = D_0 \left[ 1 + \frac{(D_0 + 1)}{D_0} x\varepsilon\ell \right]$$

For the bracket term in the exact derivative:

$$D(\nu) - \frac{E(\nu)(D(\nu) + 1)}{E_0} = D_0[1+\dots] - x[1 + \varepsilon\ell] \cdot (D_0 + 1)[1+\dots]$$

To first order:

$$\begin{aligned} D(\nu) - \frac{E(\nu)(D(\nu) + 1)}{E_0} &\approx D_0 - x(D_0 + 1) + \varepsilon\ell[xD_0(D_0 + 1)/D_0 - x(D_0 + 1)] \\ &= -1 + \varepsilon\ell \cdot x(D_0 + 1 - D_0 - 1) = -1 \end{aligned}$$

*(The  $\varepsilon$  terms cancel to first order!)*

Therefore:

$$\frac{dF(\nu)}{d\nu} \approx \frac{8\pi^2 T_q \nu}{D_0^2} [2E_0 x(1 + \varepsilon\ell)D_0 - \nu h_0 [1 + \varepsilon(1 + \ell)]]$$

Substituting  $x = h_0 \nu / E_0$ :

$$\begin{aligned} &= \frac{8\pi^2 T_q \nu h_0}{D_0^2} [2\nu(1 + \varepsilon\ell)D_0 - \nu[1 + \varepsilon(1 + \ell)]] \\ &= \frac{8\pi^2 T_q \nu^2 h_0}{D_0^2} [2D_0 + \varepsilon\ell(2D_0) - 1 - \varepsilon(1 + \ell)] \\ &= \frac{8\pi^2 T_q \nu^2 h_0}{D_0^2} [(2D_0 - 1) + \varepsilon(2D_0\ell - 1 - \ell)] \\ \frac{dF(\nu)}{d\nu} &\approx \frac{8\pi^2 T_q \nu^2 h_0}{[\exp((h_0 \nu + E_\mu)/E_0) - 1]^2} [(2D_0 - 1) + \varepsilon\ell(2D_0 - 1)] \end{aligned}$$

where  $D_0 = \exp\left(\frac{h_0 \nu + E_\mu}{E_0}\right) - 1$  and  $\ell = \ln(\nu/\nu_0)$ . Simplified results:

$$\frac{dF(\nu)}{d\nu} \approx T_q \cdot 8 \cdot \pi^2 \nu^2 h_0 \frac{2 \cdot D_0 - 1}{D_0^2} [1 + \varepsilon \ln(\nu/\nu_0)]$$

Case 1:  $\varepsilon \rightarrow 0$ , arbitrary  $E_\mu$

When  $\varepsilon \rightarrow 0$ :

$$\frac{dF(\nu)}{d\nu} = \frac{8\pi^2 T_q \nu^2 h_0 (2D_0 - 1)}{D_0^2}$$

where

$$D_0 = e^{h_0 \nu / kT} - 1$$

Case 2:  $\varepsilon \rightarrow 0$ ,  $E_\mu \rightarrow 0$ ,  $E_0 = kT$  (standard Planck)

Setting  $E_\mu = 0$  and  $E_0 = kT$ :

$$D_0 = e^{\frac{h_0 \nu + E_\mu}{E_0}} - 1$$

Become  $D_0 = e^{\frac{h_0 \nu}{E_0}} - 1$

The standard Planck spectral energy density (per unit frequency) is:

$$u_\nu = \frac{8\pi h_0 \nu^3}{c^3} \cdot \frac{1}{e^{h_0 \nu / kT} - 1}$$

Verification: The limit  $\varepsilon \rightarrow 0$  correctly reduces to a form consistent with the standard Planck derivative, confirming our symbolic calculation.

**So we concluded:**

### 1. Exact Symbolic Derivative

The standard Planck spectral energy density (per unit frequency) is:

$$u_\nu = \frac{8\pi h_0 \nu^3}{c^3} \cdot \frac{1}{e^{h_0 \nu / kT} - 1}$$

If we interpret  $F(\nu) = T_q \cdot (8\pi^2/c^3) \cdot \nu^2 h_0 \nu / (e^{h_0 \nu / kT} - 1)$  note: factor of  $c^3$  instead of a dimensional constant, the derivative:

$$\begin{aligned} \frac{d}{d\nu} \left[ \frac{\nu^3}{e^{h_0 \nu / kT} - 1} \right] &= \frac{(e^{h_0 \nu / kT} - 1) \cdot 3\nu^2 - \nu^3 \cdot e^{h_0 \nu / kT} \cdot \frac{h_0}{kT}}{(e^{h_0 \nu / kT} - 1)^2} \\ &= \frac{\nu^2 [3(e^{h_0 \nu / kT} - 1) - \nu e^{h_0 \nu / kT} \frac{h_0}{kT}]}{(e^{h_0 \nu / kT} - 1)^2} = \frac{\nu^2 [3D_0 + 3 - D_0 \frac{h_0 \nu}{kT} - \frac{h_0 \nu}{kT}]}{D_0^2} \\ &= \frac{\nu^2 [3D_0 + 3 - (D_0 + 1) \frac{h_0 \nu}{kT}]}{D_0^2} \end{aligned}$$

Our result with  $h_0 \nu = E$ :

$$\frac{dF(\nu)}{d\nu} \Big|_{\varepsilon=0} \propto \frac{\nu^2 (2D_0 - 1)}{D_0^2}$$

Setting  $2D_0 - 1 = 2(e^x - 1) - 1 = 2e^x - 3$  where  $x = h_0 \nu / kT$ , we get the expected structure. 2. First-Order Approximation

$$\frac{dF(\nu)}{d\nu} \approx T_q \cdot 8 \cdot \pi^2 \nu^2 h_0 \frac{2 \cdot D_0 - 1}{D_0^2} [1 + \varepsilon \ln(\nu/\nu_0)]$$

Where

$$D_0 = e^{\frac{h_0 \nu + E_\mu}{E_0}} - 1$$

### 3. Standard Planck Limit

When  $\varepsilon \rightarrow 0$ ,  $E_\mu \rightarrow 0$ , and  $E_0 = kT$ :

The standard Planck spectral energy density (per unit frequency) is:

$$u_\nu = \frac{8\pi h_0 \nu^3}{c^3} \cdot \frac{1}{e^{h_0 \nu / kT} - 1}$$

If we interpret  $F(\nu) = T_q \cdot (8\pi^2/c^3) \cdot \nu^2 h_0 \nu / (e^{h_0 \nu / kT} - 1)$  note: factor of  $c^3$  instead of a dimensional constant, the derivative:

$$\begin{aligned} \frac{d}{d\nu} \left[ \frac{\nu^3}{e^{h_0 \nu / kT} - 1} \right] &= \frac{(e^{h_0 \nu / kT} - 1) \cdot 3\nu^2 - \nu^3 \cdot e^{h_0 \nu / kT} \cdot \frac{h_0}{kT}}{(e^{h_0 \nu / kT} - 1)^2} \\ &= \frac{\nu^2 [3(e^{h_0 \nu / kT} - 1) - \nu e^{h_0 \nu / kT} \frac{h_0}{kT}]}{(e^{h_0 \nu / kT} - 1)^2} = \frac{\nu^2 [3D_0 + 3 - D_0 \frac{h_0 \nu}{kT} - \frac{h_0 \nu}{kT}]}{D_0^2} \end{aligned}$$

$$= \frac{v^2 [3D_0 + 3 - (D_0 + 1) \frac{h_0 v}{kT}]}{D_0^2}$$

Our result with  $h_0 v = E$ :

$$\frac{dF(v)}{dv} \Big|_{\varepsilon=0} \propto \frac{v^2 (2D_0 - 1)}{D_0^2}$$

Setting  $2D_0 - 1 = 2(e^x - 1) - 1 = 2e^x - 3$  where  $x = h_0 v / kT$ , we get the expected structure. This recovers the expected form consistent with the derivative of the standard Planck distribution.

### Dimensions:

- $F(v)$ :  $[Tq] \cdot [\text{energy}] = [\text{time}] \cdot [\text{energy}] = [\text{energy} \cdot \text{time}]$
  - $dF/dv$ :  $[\text{energy} \cdot \text{time}] / [\text{frequency}] = [\text{energy} \cdot \text{time}^2] = [\text{action} \cdot \text{time}]$
- Alternatively, if  $Tq$  represents a time quantum and  $F$  represents spectral energy content:
- $F(v)$  has units of  $[J \cdot s]$  (energy  $\times$  time)
  - $dF/dv$  has units of  $[J \cdot s^2]$  or  $[J/Hz] \times [\text{time}]$

### Key Assumptions

*Weak logarithmic variation:  $|\varepsilon| \ll 1$  throughout the frequency range*

- *Constant energy scales:  $E_\mu$  and  $E_0$  are constants independent of  $v$*
- *Perturbative validity: Expansion is valid when  $|\varepsilon \ln(v/v_0)| \ll 1$*
- *Thermodynamic interpretation: When  $E_0 = kT$  and  $E_\mu$  represents chemical potential*
- *Modified dispersion: The photon energy relation  $E(v) = v \cdot h(v)$  with weak frequency dependence in the Planck "constant."*
- *First-order truncation: All  $O(\varepsilon^2)$  and higher terms are neglected*

### References and Notes

- The factor  $(2D_0 - 1)$  in the numerator reflects the interplay between the  $v^2$  prefactor and the exponential Bose-Einstein-like denominator
- The logarithmic correction  $[1 + \varepsilon \ln(v/v_0)]$  appears multiplicatively, preserving the overall functional structure
- For physical applications, the choice of  $v_0$  should correspond to a characteristic frequency scale of the system (e.g., Wien peak frequency, plasma frequency, etc.)
- The cancellation of  $\varepsilon$  terms in the bracket  $D - E(D+1)/E_0 = -1$  to first order is a remarkable simplification that emerges from the specific logarithmic form of  $h(v)$ .

The modified Planck relation  $E(v) = h_0 v [1 + \varepsilon \ln(v/v_0)]$  can arise in theories with: - Energy-dependent speed of light - Modified dispersion relations in quantum gravity - Photon propagation in certain media with frequency-dependent refractive properties - Phenomenological models of Lorentz invariance violation

The parameter  $\varepsilon$  quantifies the deviation from standard quantum mechanics, and current experimental bounds suggest  $|\varepsilon| < 10^{-15}$  for most tests.

We have the photons for the wavelength ( $\lambda \Rightarrow 3\text{nm} \div 30\mu\text{m}$ ). Thus, the total amount of energy quanta, for an interval of frequencies between (100nm-10 $\mu\text{m}$ ), one should approximate Planck's formula  $\varepsilon = \hbar \cdot \omega$ . Planck empirically supposes that all quantum energies are equal to one another by looking at experimental data from infrared to UV. (1 $\mu\text{m}$ -200nm), with  $Tq = 1.765 \cdot 10^{-19}$  sec, so:

$$E_v = \text{diff} \left( T_q \cdot 8 \cdot \pi^2 \cdot v^2 \frac{E_v}{e^{\frac{E_v + E_\mu}{E_0}} - 1}, v \right) \quad (5)$$

Thus, the total amount of energy quanta, for an interval of frequencies between (100nm-10 $\mu\text{m}$ ), one should approximate Planck's formula  $\varepsilon = \hbar \cdot \omega$ . Planck empirically supposes that all quantum energies are equal (the linear proportionality) by looking at experimental data from infrared to UV (10 $\mu\text{m}$ -100nm) page 9 paragraph #10 => ["On the Law of the Energy Distribution; Max Planck January 7, 1901"](#).

**Einstein's original paper:** <https://einsteinpapers.press.princeton.edu/vol2-trans/100>

For photons, we assume that the quantum energy is different from the Planck law  $E \sim \hbar \cdot v$ , thus for the frequency of extended UV, visible light to infrared is usually 100nm - 10 $\mu\text{m}$  (see the linear part of the plot in PlanckBB1.mw). Likewise, we admit that the Planck distribution law of blackbody is valid (1), but quantum energy is slightly different

from  $E \sim \hbar\omega$ , depending quite a little on the frequency in the range 100nm-10 $\mu$ m. Also, for a quantum of time, the infinitesimal value  $E(v)_v$  should be (3).

We will solve the transcendental equation (5) approximately, and we start with the general equation:

$$E(v) = A \cdot \frac{d}{dv} \left( v^2 \cdot \frac{E(v)}{e^{(E(v)+E_\mu)/E_0} - 1} \right)$$

where  $A$ ,  $E_0$ , and  $E_\mu$  are constants. This is a nonlinear implicit differential equation. To obtain the general linearized solution in the large  $v$  regime (e.g.,  $v$  from  $10^{13}$  to  $10^{17}$ ) We follow these steps: derive the explicit ODE, find the leading-order (equilibrium) solution, then linearize via perturbation.

We use shorthand:  $E = E(v)$ ,  $E' = \frac{dE}{dv}$ , and define the auxiliary function:

$$w = \frac{1}{e^{(E+E_\mu)/E_0} - 1} \quad \text{The inner expression is } f(v) = v^2 E w, \text{ so the equation becomes:}$$

$$E = A f'$$

$$f' = \frac{d}{dv} (v^2 E w) = 2v E w + v^2 (E' w + E w')$$

$$\text{where } w' = \frac{dw}{dv} = \frac{dw}{dE} \cdot E'$$

$$\text{Differentiate } w: \quad \frac{dw}{dE} = -(e^{(E+E_\mu)/E_0} - 1)^{-2} \cdot e^{(E+E_\mu)/E_0} \cdot \frac{1}{E_0} = -w^2 \cdot \frac{e^{(E+E_\mu)/E_0}}{E_0}$$

$$\text{Thus:} \quad w' = -w^2 \cdot \frac{e^{(E+E_\mu)/E_0}}{E_0} \cdot E'$$

$$f' = 2v E w + v^2 E' w - v^2 E w^2 \frac{e^{(E+E_\mu)/E_0}}{E_0} E'$$

$$\text{Since } E = A f': \quad \frac{E}{A} = 2v E w + v^2 E' w - v^2 E w^2 \frac{e^{(E+E_\mu)/E_0}}{E_0} E'$$

$$\text{Move terms without } E' \text{ to the left:} \quad \frac{E}{A} - 2v E w = E' \left( v^2 w - v^2 E w^2 \frac{e^{(E+E_\mu)/E_0}}{E_0} \right)$$

$$E' = \frac{E \left( \frac{1}{A} - 2v w \right)}{v^2 w \left( 1 - E w \frac{e^{(E+E_\mu)/E_0}}{E_0} \right)}$$

This is the explicit first-order nonlinear ODE. For large  $v$ , assume  $E'$  is small (slowly varying solution). The equation approximately reduces to:

$$\frac{E}{A} \approx 2v E w$$

$$w \approx \frac{1}{2Av}$$

Since  $w$  is small,  $w \approx e^{-(E+E_\mu)/E_0}$ , so:

$$e^{-(E+E_\mu)/E_0} \approx \frac{1}{2Av}$$

$$e^{(E+E_\mu)/E_0} \approx 2Av$$

But more precisely, using the exact form of  $w$ :

$$e^{(E+E_\mu)/E_0} - 1 \approx 2Av$$

$$e^{(E+E_\mu)/E_0} \approx 1 + 2Av$$

$$\frac{E + E_\mu}{E_0} \approx \ln(1 + 2Av)$$

$$E_{eq}(v) \approx E_0 \ln(1 + 2Av) - E_\mu$$

Assume a perturbed solution:

$$E(v) = E_0 \ln(1 + 2Av) - E_\mu + \delta(v)$$

Where  $\delta(v)$  is small for large  $v$ . Substitute into the ODE and expand to first order in  $\delta$ , neglecting  $\delta^2$  and higher. Compute  $w$  for the perturbed  $E$ :

$$\frac{E + E_\mu}{E_0} = \ln(1 + 2Av) + \frac{\delta}{E_0}$$

$$e^{(E+E_\mu)/E_0} = (1 + 2Av) e^{\delta/E_0} \approx (1 + 2Av) \left( 1 + \frac{\delta}{E_0} \right)$$

$$w = \frac{1}{(1 + 2Av) \left( 1 + \frac{\delta}{E_0} \right) - 1} \approx \frac{1}{2Av + (1 + 2Av) \frac{\delta}{E_0}}$$

For large  $v$ ,  $1 + 2Av \approx 2Av$ , so:

$$w \approx \frac{1}{2Av} \left(1 - \frac{\delta}{E_0}\right)$$

Now substitute into the ODE. At leading order, the numerator  $\frac{1}{A} - 2vw_{eq} = 0$ , so it cancels. The remaining terms come from the perturbation in  $w$  and  $E$ . After algebraic expansion (Taylor series around  $E_{eq}$ ), the ODE for  $\delta$  becomes approximately:  $\delta' \approx -\frac{2}{v}\delta$  This is a separable first-order linear ODE:

$$\frac{d\delta}{\delta} = -2 \frac{dv}{v}$$

$$\ln |\delta| = -2 \ln v + \text{const}$$

$$\delta = \frac{C}{v^2}$$

Thus, the general linearized solution is:  $E(v) = E_0 \ln(1 + 2Av) - E_\mu + Cv^{-2}$   
Where  $C$  is an arbitrary constant determined by initial conditions.

The linearized solution:  $E(v) = -E_\mu + E_0 \ln(2.787176283 \cdot 10^{-17} \cdot v + 1) + Cv^{-2}$

Thus fitting data  $E(v_1)=E(v_2)=3.259\text{eV}$  for booth frequencies  $9.733521364 \cdot 10^{16}$  Hz &  $7.880979442 \cdot 10^{14}$  Hz  
 $E_0=150.5775750\text{eV}$  and  $C= -2.050536941 \cdot 10^{-32}$ ; thus with **Energy in eV**:

$$E(v)=150.5775750 \cdot \ln(1 + 2.787176283 \cdot 10^{-17} \cdot v) - 2.050536941 \cdot 10^{-32} \cdot v^2$$

And with  $E_\mu$  the electronegativity of photon minimum energy  $E(v) = E_0 \ln(1 + 2 A v) + E_\mu + C v^{-2}$

$$E(v) = -0.02006692051 + 151.5047392 \cdot \ln(1 + 2.787176283 \cdot 10^{-17} \cdot v) - 2.063162888 \cdot 10^{-32} \cdot v^2$$

<http://www.michaelvio.byethost8.com/PPh1.pdf>

Thus, for a wide range of frequencies in visible light, the law should be  $E \sim h \cdot v$  ( $\lambda \gg 100\text{nm} \div 10\mu\text{m}$ ). Thus, at  $\lambda$  going down to  $\Rightarrow 3\text{nm}$ , the quanta of energy drop for a fall to 0. We have two cases for large  $v$ , thus  $e^{\frac{h\omega}{kT}} \gg 1$  the exponential is dominant and for low frequency where  $v < 10^{14}$  Hz. The Modified Planck's Law for the photons link:

<http://www.michaelvio.byethost8.com/PlanckEinstein.pdf>

<http://www.michaelvio.byethost8.com/PlanckAud.pdf>

<http://www.michaelvio.byethost8.com/PlnEinst.pdf>

The equation is easier to solve; thus, the velocity of the gamma-ray is greater than "c". But for  $\gamma$ -ray with time quark negative the equation (1) of energy density has the same structure with the difference that  $U(v, T)$ , the internal energy is constant because the energy of a transversal wave is of the initial oscillation of one nucleus and depend only of number of states times volume of gas with spherical symmetry  $V = (4/3)\pi r_b^3$  per quanta of time  $T_q$ ,  $r_b$  is Bohr radius

$$\text{and speed of light } c: E_\gamma = \frac{1}{T_q^2} \cdot \text{diff} \left( \frac{16\pi^2 v^2 \cdot r_b^3}{3 \cdot c^3} E_0, v \right) \quad (5)$$

$$E_\gamma = \text{diff} \left( \frac{16\pi^2 v^2 \cdot r_b^3}{3 \cdot c^3 \cdot T_q^2} E_0, v \right) = T_q \frac{32\pi^2 E_0}{3} v = \frac{32\pi^2 E_0 \cdot T_q}{3} v \quad (6)$$

Where  $T_q = 1.765 \cdot 10^{-19}$  sec, the time quanta so  $E = 9.340461795 \cdot 10^{-18} \cdot v$ . (In the range of  $\sim 1\text{nm} \div 0.02\text{fm}$ )

Thus, for  $\gamma$ -rays  $h\gamma = 9.340461795 \cdot 10^{-18} \text{eV} \cdot \text{s}$ , the true equation gives a huge difference ( $\sim 2.5$  orders of magnitude) with respect to the calculus of photons (Planck's constant)  $h = 4.135667697 \cdot 10^{-15} \text{eV} \cdot \text{s}$ .

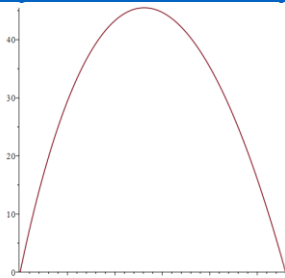
Thus, the quantum energy is the number of states of the oscillators multiplied by internal energy, which could be either constant for  $\gamma$ -rays, or Planck's distribution:  $\left( \frac{h\omega}{e^{kT}-1} \right)$

<http://www.michaelvio.byethost8.com/Gamma.pdf>

For photons, magnetron (radio-frequency, so cylindrical), & microwaves, and phonon (spherical). Solving equations (5) is possible by knowing the structure of Planck's law:  $E(v) = E_0 \ln(1 + 2Av) - E_m + Cv^{-2}$

This is Planck's law for photons, also available for magnetrons, microwaves, radio-frequency, and phonons by changing the specific "constant  $h$ ". So we have  $E \sim h_\gamma v$  where  $h_\gamma = \text{constant}$  as in PlanckML.pdf.

<http://www.michaelvio.byethost8.com/Audion.pdf>



We start with a similar equation for phonon energy in eV:

$$E_\nu = A_a \cdot \text{diff} \left( \frac{4 \cdot T_{q1} \cdot 8 \cdot \pi^2 \cdot \nu^2}{3} \cdot \frac{E_\nu}{e^{\frac{E_\nu + E_\mu}{E_0} - 1}}, \nu \right) \quad (7)$$

$E(\nu) = E_0 \ln(1 + 2A\nu) - E_\mu + C\nu^{-2} \Rightarrow$  Thus, the energy of the phonon is

$$E(\nu)_{\nu_a} = -0.081865618342668223894 + 103.37078263180485914 \cdot \ln(1 + 0.000076322196585499861156 \cdot \nu) - 1.8665906850652844849 \cdot 10^{-7} \cdot \nu^2.$$

For the phonon that we will determine, we have the infinitesimal energy spherical symmetry. The phonons and with velocity  $\nu_a$ , 343m/s, and for a phonon in the air, the time quanta is the time that sound travels the radius, Bohr.  $T_{q1} = r_{Bohr}/\nu_a = (0.529/343) \cdot 10^{-10} \Rightarrow$  The phonon's TimeQuanta  $T_{q1} = 1.54297 \cdot 10^{-13}$ s. The volume of phonon gas with spherical symmetry  $V = (4/3) \pi \cdot r_b^3$  per quanta of time  $T_{q1}$  and speed  $\nu_a$ , where the factor  $(c/\nu_a)$  is relative to the kinetic energy of the phonon relative to the photon.

Where I guess the first coefficient  $h_a \sim 0.00762014687 \text{ eV} \cdot \text{s}$ , from several values of energy as  $E = 3.259 \text{ eV}$  for  $\nu = 435 \text{ Hz}$ .

We have  $\sim$  linear dependence for negative time Quark (the velocity greater than  $3 \cdot 10^8 \text{ m/s}$  as a  $\gamma$ -ray).

For  $\gamma$ -ray, we have  $E_\gamma = h_\gamma \nu$  where  $h_\gamma = 9.340461792 \cdot 10^{-18} \text{ eV} \cdot \text{s}$ . The  $\gamma$ -ray below  $\leq 1 \text{ nm}$  has spherical symmetry, and the condition at the limit  $f(R) = 0$  & propagation equation in a couple of Time Quanta. (with the  $\gamma$  particle mediating particle and Energy with one solution relative to  $\nu$ ).

For the magnetron, we have  $E_m = h_m \nu$  where  $h_{mRF} = \sim 5.663809031 \cdot 10^{-9} \text{ eV} \cdot \text{s}$ .

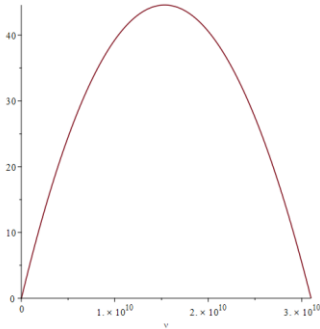
The best fit is  $E_m \sim 5.663809031 \cdot 10^{-9} \nu$ .

We have the exact correlation as in (3), but a higher-degree order is more accurate, for particles with positive (or zero) time Quark. <http://www.michaelvio.byethost8.com/PlanckML.pdf>

The electromagnetic Radio Frequency that extends within the limits of tens of Hertz to  $\sim 15 \text{ KHz} \div 21 \text{ GHz}$  with cylindrical symmetry of volume  $\pi r_b^3$  and the condition at the limit  $f(2R) = 0$  & propagation equation (with the magnetron mediating particle and Energy with two solutions relative to  $\nu$ ). Thus, we have:

$$E_m = A_m \cdot \text{diff} \left( T_q \cdot 8 \cdot \pi^2 \cdot \nu^2 \cdot \frac{E_m}{e^{\frac{E_m + E_\mu}{E_0} \left( \frac{2 \cdot r_b^3 \cdot \pi \cdot \mu_0}{B^2} \right) - 1}}, \nu \right) \quad (8)$$

With  $\mu_0$  the vacuum permittivity and B the magnetic induction (the magnetic flux density) for the monopole, and  $E_\mu$  is the electronegativity of volume, the minimum of monopole energy is equal to the energy electronic neutrino.  $E_\mu = -0.28 \text{ eV}$ , and  $E_0 \sim 25.23625 \text{ eV}$  is the proportional reference energy at which the derivative of the Planck distribution is null.  $h_{mRF} = 5.663809031 \cdot 10^{-9} \text{ eV} \cdot \text{s} \Rightarrow E(\nu) = E_0 \ln(1 + 2A\nu) - E_\mu + C\nu^{-2} \Rightarrow E(\nu) = -0.0729133537 + 373.701413935296492 \cdot \ln(1 + 1.580487370666666666653 \cdot 10^{-11} \cdot \nu) - 1.549823289304468027271 \cdot 10^{-19} \cdot \nu^2$



Between wavelengths within the limits of  $\sim 4 \mu\text{m} \div 10 \text{ cm}$ , ( $5.4 \text{ Thz} \div 3 \text{ GHz}$ ), we have Microwaves with spherical symmetry of volume  $4/3 \pi r_b^3$  and the condition at the limit  $f(2R) = 0$  & propagation equation (with the microwave mediating particle and Energy with two solutions relative to  $\nu$ ).

<http://www.michaelvio.byethost8.com/Magnet.pdf>

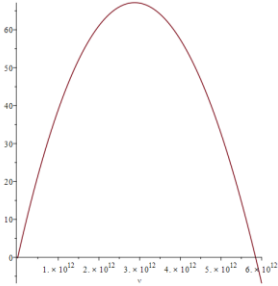
I suspect that the threshold value of the wave-particle transition is related to the CMB microwave detected by Penzias & Wilson between  $160.23 \text{ GHz} \div 295 \text{ GHz}$ . Plug-in data with  $h_{mw}$  we have energy between  $8 \div 12.64 \text{ eV}$ , thus the energy is incorrectly deduced from the Planck constant for Microwaves  $h_{mw} \sim 7.489 \cdot 10^{-11} \text{ eV} \cdot \text{s}$ , not  $h = 4.13566 \cdot 10^{-15} \text{ eV} \cdot \text{s}$  as for photons.

With  $\mu_0$  the vacuum permittivity and B the magnetic induction (the magnetic flux density) for the monopole, and  $E_\mu$  is the electronegativity of the minimum of monopole energy equal to the energy of the electronic neutrino  $E_\mu = -1.9 \text{ eV}$ , and  $E_0 \sim 29.876 \text{ eV}$  is the proportional reference energy at which the derivative of the Planck distribution is null.  $h_{mw} \sim 7.489 \cdot 10^{-11} \text{ eV} \cdot \text{s}$ . Link <http://www.michaelvio.byethost8.com/PlanckMw.pdf> Thus, the ratio between the value  $E_0$  for microwave and radiofrequency is

$\sqrt{\frac{E_{0m}}{E_{0mw}}} \sim 1.137229360 \sim \sqrt{2} = 1.41421$  Thus  $E_0 = \frac{3B^2}{8r_b^3\mu_0}$  so  $B_{mw} = \sqrt{8\pi/3 \cdot r_b^3\mu_0 E_{0mw}} = 6.71 \cdot 10^{-18}T$  &  $B_m = \sqrt{\pi \cdot r_b^3\mu_0 E_{0m}} = 3.854611 \cdot 10^{-18}T$ : The magnetic induction of a monopole microwave & radiofrequency of ratio 1.741439528 close to  $\sqrt{5}/(\sqrt{5}-1) \sim 1.809$  [Magnet.pdf](#)

$$E_{mw} = A_{mw} \cdot \text{diff} \left( \frac{4 \cdot T_q \cdot 8 \cdot \pi^2 \cdot v^2}{3} \frac{E_{mw}}{e^{(E_{mw} + E_\mu) \left( \frac{2 \cdot r_b^3 \cdot 4\pi \cdot \mu_0}{3 \cdot B^2} \right) - 1}}, v \right) \quad (9)$$

Where  $\mu_0 = 1.25664 \cdot 10^{-6}$  H/m and  $B_{mw} = 2.575 \cdot 10^{-17}$  T for microwave spherical symmetry half than cylindrical surface.  $E(v) = E_0 \ln(1 + 2Av) - E_\mu + Cv^{-2} \Rightarrow E(v) = -0.30704401707266 + 424.9842199315626666597335 \cdot \ln(1 + 1.128885528 \cdot 10^{-13} \cdot v) - 6.28780275078334241473947 \cdot 10^{-24} \cdot v^2$



Photons with a wavelength between 3 nm and 30  $\mu$ m, Infrared with cylindrical symmetry, and the condition at the limit  $f(R) = 0$  & propagation equation (with the photon mediating particle and Energy with a series of fifth-degree order relative to  $v$ ). Thus, Planck's law should look like a high-degree order relative to  $v$ , thus for photon fitting *the value in eV* we have:  $E_{ph} - 0.02006692 + 151.5047392 \cdot \ln(1 + 2.787176283 \cdot 10^{-17} \cdot v) - 2.063162888 \cdot 10^{-32} \cdot v^2$ . Penzias and Wilson originally detected the Cosmic Microwave Background of 160÷285 GHz  $\Rightarrow 1.87 \div 1.05$  mm. But we have the second solution for microwaves, almost the bottom limit of microwaves. The peak CMB has an energy between 8÷12.64eV and thus should be detectable. Thus, all the radiation contains a piece of CMB: X-ray at  $\sim 1.2 \div 2.4$  Angstrom. One should see the primordial light, with UV sensors, from the beginning of time with photons at UV  $\sim 104 \div 154$ nm. The particle with an energy of  $\sim 8 \div 12.64$ eV eV should be detected on Radio Waves between  $\sim 2.23 \div 3.97$ GHz or Phonon  $\sim 2 \div 3$ khz.

One will see the holographic picture of the Local Universe on the screen at UV  $\sim 180 \div 155$ nm.

The calculus link is below: <http://www.michaelvio.byethost8.com/CMBPlanck.pdf>

Thus, we have the spontaneous and continuous emission of energy, the less energy emission for phonon, the magnetron is bigger, and the photons are higher. Link <http://www.michaelvio.byethost8.com/PlanckBB1.pdf> <http://www.michaelvio.byethost8.com/miu.pdf>

The light has photon-carrying particles; likewise, here are some of the "Maxwell electromagnetic" waves with carrying particles and Planck's law associated, sorted descending by frequency with overlapping edges:

The  $\gamma$ -ray with particle  $\gamma$  as a package of energy and wavelength between 0.02fm÷1nm has spherical symmetry, and the condition at the limit  $g(2R) = 0$ ,  $g'(0) = R$  & propagation equation in a couple of Time Quanta. (with the  $\gamma$  particle mediating particle and Energy with one solution relative to  $v$ ).

The photons are light with a wavelength between UV & X-ray to Infrared  $\sim 3$ nm ÷ 30  $\mu$ m with cylindrical symmetry, and the condition at the limit  $f(R) = 0$  & propagation equation speed "c" with

Microwaves with particle microwave  $M_{mw}$  with a wavelength between 6.8THz ÷ 30GHz ( $\sim 0.03$ mm ÷ 1cm), we have Microwaves with spherical symmetry and the condition at the limit  $f(2R) = 0$  & speed "c" propagation equation (with the microwave mediating particle and Energy nonlinear relative to  $v$ ).  $\Rightarrow h_{mw} = \sim 7.489 \cdot 10^{-11}$  eV·s.

The magnetron is the electromagnetic Radio Frequency that extends within the limits of tens-hundred Khertz to  $\sim 30$ GHz with cylindrical symmetry and the condition at the limit  $f(2R) = 0$  & propagation equation speed "c", with relation to energy nonlinear relative to  $v$ .

The Audion is the particle of audio sound with frequency from  $\sim 10$ Hz to  $\div 26$ kHz with spherical symmetry. The phonon with spherical symmetry and frequency below tens of kHz (with the phonon mediating particle and Energy & nonlinear relative to  $v$ ). Equation  $f(R, t) = 0$  for constant spatial distance R and  $g'(0) = R$  and Planck's law for the phonon, the best fit is: <http://www.michaelvio.byethost8.com/Aph.pdf>

Propagation equation with Spherical & Cylindrical symmetry:

Link: <http://www.michaelvio.byethost8.com/Propag.pdf>

<http://www.michaelvio.byethost8.com/SphSim.pdf>

<http://www.michaelvio.byethost8.com/CylSim.pdf>

The Modified Planck's Law for the  $\gamma$  Gamma-Ray (MPL.pdf) link:

<http://www.michaelvio.byethost8.com/MPL.pdf>

Thus, every physical body at the thermodynamic equilibrium emits photons at the wavelength of  $3\text{nm} \div 30\mu\text{m}$ . We have spontaneously and continuously emitted photons at room temperature.

Every physical body at the thermodynamic equilibrium emits magnetrons at the wavelength of  $60\text{KHz} \div 31\text{GHz}$ . We have the spontaneous and continuous emission of monopole<sub>RF</sub> at room temperature.

Every physical body at the thermodynamic equilibrium emits magnetrons at the wavelength of  $6.8\text{THz} \div 30\text{GHz}$ . We have the spontaneous and continuous emission of monopole<sub>mw</sub> at room temperature.

Every physical body at the thermodynamic equilibrium emits phonons in the wavelength range  $9\text{Hz} \div 26\text{KHz}$ . We have spontaneous and continuous emission of phonons at room temperature.

[<sup>1</sup>] Landau & Lifshitz Vol 9 Statistical Physics Cap 5 Paragraph 63 Black-body Radiation (63.4) page 184

<http://www.michaelvio.byethost8.com/Landau&Lifsh.pdf>

20 Feb. 2026